Subject (L.02) Date.

Remember that :-

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{b-a}{n} f\left(a + \frac{b-a}{n} k\right).$$

Thearm :-
Pg 80

If $f(x)$ is a continous fn on $[a,b]$, then there exits a point $c \in [a,b]$

S.t $$\boxed{\frac{1}{b-a} \int_a^b f(x)\,dx = f(c)}$$

Some properties of Delimite Integral :-
If $f(x)$, $g(x)$ are continous fns on $[a,b]$
$d \in [a,b]$, const

1- $\int_a^b [F(x) \pm g(x)]\, dx = \int_a^b F(x)\, d(x) + \int_a^b g(x)\, dx$

2- $\int_a^b c\, F(x)\, dx = c \int_a^b F(x)\, dx$

3- $\int_a^b F(x)\, dx = \int_a^d F(x)\, d(x) + \int_a^b F(x)\, dx$

4- $\int_a^b F(x)\, dx = - \int_b^a F(x)\, d(x)$

5- $\int_a^a F(x)\, dx = \text{Zero}$

6- If $F(x) \leq g(x) \Rightarrow \int_a^b F(x) \leq \int_a^b g(x)$

$\forall$ all $x \in [a, b]$

F(x) odd function

7- $\int_{-a}^{+a} F(x)\, d(x) = \text{zero}$:

F(x) even

8- $\int_{-a}^{+a} F(x)\, dx = 2 \int_0^a F(x)\, dx$

Theorem ②:-

Let $f(x)$ be a continous Fn on $[a, b]$

If $x \in [a, b]$

and $\int_a^x f(x)\, dx = F\,X$

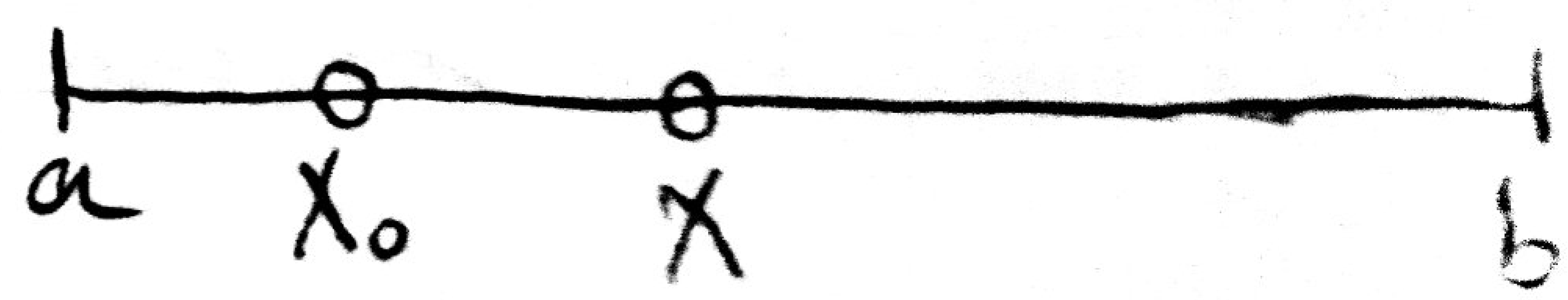
$F(x)$ must be a continous on $[a, b]$

and differentiable on (a, b)

, then ??

$$\boxed{F'(x) = f(x)}$$

prove of theorem 2



$$\frac{F(x) - F(x_0)}{x - x_0}$$

$$= \frac{1}{x - x_0} \left\{ \int_a^x f(x)\, dx - \int_a^{x_0} f(x)\, d(x) \right\}$$

$$= \frac{1}{x - x_0} \int_{x_0}^{x} f(x)\, dx$$

$$= f(c) \quad , c \in [x_0, x]$$

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taking the Limit

as $X \longrightarrow X_0$

then We get :-

$$F'(X_0) = \lim_{X \to X_0} F(c) = F(X_0)$$

Since X_0 is arbitrary point $\in [a, b]$. Then :-

$$F'(X) = f(X) \quad \forall \text{ all } X \in [a, b]$$

Theorem :

the Fundamental theorem of calculus :-

$$\int_a^b f(X) = F(b) - F(a)$$

$$\int_1^4 X \, dX = \frac{X^2}{2} \Big|_1^4 = \frac{4^2}{2} - \frac{1^2}{2}$$

$$\boxed{\int f(X) \, dX = F(X) + C}$$

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EX ② Pg ②

(1) ؟ $\frac{d}{dX}[X^2e^X]$

Sol:

$[X^2e^X]' = 2Xe^X + X^2e^X \rightarrow (1)$ هامة :-

(2) ؟ $\int[2Xe^X + X^2e^X]dX = X^2e^X + C \longrightarrow (2)$

(3) ؟ $\int_1^5[2Xe^X + X^2eX]dX = X^2e^X\Big|_1^5 = 5^2e^5 - 1^2e^1 = 25e^5 - e$

$= 3507.6$

$$\int x^n dX = \frac{X^{n+1}}{(n+1)} + C \quad , n \neq -1$$

$$\int x^{-1} dX = \ln X + C$$

$\boxed{\text{table Pg 566} \Leftarrow \text{حفظ}}$

$$\int a^X dX = \frac{a^X}{\ln a} + C$$

$$\int a^{bX} dX = \frac{a^{bX}}{b\ln a} + C$$

Pg 5:-

4- $\int \sin(aX)\,dX = -\frac{1}{a}\cos(aX) + C$

6- $\int \cos(aX)\,dX = \frac{1}{a}\sin(aX) + C$

8- $\int Sec^2(aX) = \frac{1}{a}\tan(aX) + C$

10- $\int Cosec^2(aX) = -\frac{1}{a}\cot(aX) + C$

17- $\int \frac{1}{1+X^2}\,dX = \tan^{-1}X + C$

18- $\int \frac{1}{a^2+X^2}\,dX = \frac{1}{a}\tan^{-1}\left(\frac{X}{a}\right) + C$

ex.5 $\qquad [a^u]' = u'\,a^u \ln a$

Pg 8

$I = \int \left[t^{\frac{2}{3}} + \frac{4}{5+t^2} - 3^{(2t)}\right] dt$

$I = \underbrace{\int t^{\frac{2}{3}}\,dt}_{I_1} + \underbrace{\int \frac{4}{5+t^2}\,dt}_{I_2} - \underbrace{\int 3^{2t}\,dt}_{I_3}$

$I_1 = \int t^{\frac{2}{3}}\,dt = \frac{3t^{\frac{5}{3}}}{5}$

$I_2 = 4\int \frac{1}{(\sqrt{5})^2+t^2}\,dt = \frac{4}{\sqrt{5}}\tan^{-1}\frac{t}{\sqrt{5}}$

$I_3 = \int 3^{2t} = \frac{1}{2\ln 3}\int 2\times 3^{2t}\ln 3\,dt = \frac{1}{2\ln 3} \times 3^{2t}$

$I = \frac{3}{5}t^{\frac{5}{3}} + \frac{4}{\sqrt{5}}\tan^{-1}\frac{t}{\sqrt{5}} + \frac{1}{2\ln 3} \times 3^{2t} + C$

Find Pg. 9

a) $\int \frac{x^2+1}{x}\, dx$

b) $\int \sin^4 x\, dx$

Sol

a) $I = \int \frac{x^2+1}{x}\, dx = \int \left(\frac{x^2}{x} + \frac{1}{x}\right) dx$

$= \int x\, dx + \int \frac{1}{x}\, dx = \frac{x^2}{2} + \ln|x| + C$

b) $\int \sin^4 x\, dx$

Remember that 2010

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$

$\sin^4 x = \frac{1}{4}[1 - \cos 2x]^2 = \frac{1}{4}[1 - 2\cos 2x + \cos^2 2x]$

$\cos^2 x = \frac{\cos 2x + 1}{2}$

$\cos^2 2x = \frac{\cos 4x + 1}{2}$

$\sin^4 x = \frac{1}{4}\left[1 - 2\cos 2x + \frac{\cos 4x + 1}{2}\right]$

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$$\int \sin^4 x \, dx = \int \frac{1}{4}\left[1 - 2\cos 2x + \frac{\cos 4x + 1}{2}\right] dx$$

$$= \frac{1}{4}\left[x - 2\frac{\sin 2x}{2} + \frac{1}{2}x + \frac{1}{2}\frac{\sin 4x}{4}\right] + C$$